Internal Flow Mechanism in Filter Cakes

MOMPEI SHIRATO, MASAO SAMBUICHI, HIROO KATO, and TSUTOMU ARAGAKI

Nagoya University, Nagoya, Japan

The definition of filtration resistance is modified by considering relative solid-liquid velocity. The internal flow mechanism in a filter cake is reexamined in view of the movement of solids during compression. Under conditions of short filtrations involving highly concentrated slurries, the velocity of solids is shown to be comparable to the velocity of the liquid. A differential equation is proposed for the flow through compressible cakes in which the pressure gradient is assumed proportional to the difference in average velocities of the liquid and solid rather than to the average velocity of the liquid alone.

An improved definition of the average filtration resistance is developed on the basis of the new flow equation.

Development of filtration theory in recent years has been based upon differential equations involving local flow resistances and variable flow rates. The local filtration resistances have been related to experimental values of compressive pressure by means of the compression-permeability cell as designed by Carman (*1*) and Ruth (*7*). Recent work by Tiller and co-workers (*13* to *15*) has led to a better understanding of the internal flow mechanism. Tiller and Cooper (*13*) developed a differential equation involving a variable fluid flow rate q_x. However, they failed to recognize that the movement of the solids was such that the solid velocity could not always be assumed to have a value of zero. This paper demonstrates how the basic flow equation can be modified to take into account the average liquid velocity relative to the average solid velocity.

In Figure 1, a schematic diagram of a cake is shown. As the cake is compressed, the porosity decreases with time at a given distance, x, from the medium. Decrease in porosity is caused by solid flowing into the voids as the cake compresses. The compressive, squeezing action causes the flow rate of the liquid to increase as the medium is approached (*5, 9, 10, 13* to *15*).

In Figure 2, plots of the porosity ϵ_x vs. x are shown in relation to cake thickness L at time θ and $(L + dL)$ at time $(\theta + d\theta)$. The liquid volume squeezed out from the cake during time $d\theta$ is represented by the area MBE, the liquid volume added to the cake by ABED, and the net solid volume is MBCFE. The liquid volume squeezed out of the cake exactly equals the displaced liquid.

Variable flow rates of liquid and solid are of importance in short filtrations involving concentrated slurries as often practiced in continuous, vacuum filters and filter presses. Ordinary filtration equations as used by many investigators (*1, 2, 4, 7*) can be in error if the slurry is highly concentrated. Errors ranging from 5 to 25% may be encountered if the velocity variations are neglected.

The percentage of voids in filter cakes varies widely. In some cases a cake will contain less than 5% solids by volume; while in other cases, it may run as high as 70% solids. In general for filtration, the slurry must contain less solids than the cake. In fact, the slurry will have a smaller percentage of solids than the surface of the cake where the percentage of solids $(1/m_i)$ is a minimum. Thus the slurry concentration s must be less than $1/m_i$, if the slurry is not, in fact, a cake. A concentrated slurry is one in which s approaches $1/m_i$. It is not possible to define a precise limit for concentrated slurries because of the differences in behavior of different materials.

Tiller and Shirato (*15*) defined a new average filtration resistance on the basis of variable flow rate throughout the cake. However, they failed to take the variation of solid velocity into account. Further modification and improvement on their methods is made in this paper by considering the velocity of the liquid relative to the solids.

BASIC DIFFERENTIAL EQUATION OF FILTRATION

The basic differential equation for filtration has generally been presented in the form (*15*).

$$g_c \frac{dp_s}{dw_x} = \frac{g_c}{\rho_s(1 - \epsilon_x)} \frac{dp_s}{dx} = -\mu \alpha_x q_x \tag{1}$$

where p_s is the solids compressive pressure and q_x is the apparent flow rate of liquid at a distance x from the medium as shown in Figure 1. Equation (1) rests upon the assumption that the liquid moves past stationary solid particles. The solids move toward the septum as the cake is compressed during filtration, and it is false to assume that the solid velocity is zero. As a practical matter the velocity of the solids is important for highly concentrated slurries. Equation (1) must be modified where the velocity of the solids is comparable to the velocity of the liquid. A concentrated slurry is roughly defined as one in which the solid content in the slurry contains about 50 to 75% of the solid concentration at the cake surface ($m_i s$ is greater than 0.5).

It should be recognized that the internal flow rates of liquid, q_x, and solids, r_x, are not constant due to the continuous compression of the cake.

If ϵ_x is the porosity at x, the true average velocity of the liquid is represented by q_x/ϵ_x and the true average velocity of the solids is given by $r_x/(1-\epsilon_x)$. Therefore the true average relative velocity, u_x, of liquid to solids is represented by

$$u_x = \frac{q_x}{\epsilon_x} - \frac{r_x}{1-\epsilon_x} \tag{2}$$

Multiplying Equation (2) by the local porosity, ϵ_x, yields the apparent relative flow rate of liquid to solids based upon unit cross-sectional area

$$\epsilon_x u_x = q_x - \frac{\epsilon_x}{1-\epsilon_x} r_x = q_x - e_x r_x \tag{3}$$

where e_x is the local void ratio. Replacing the flow rate of the liquid by the flow rate relative to the solids in Equation (1) leads to

$$g_c \frac{dp_s}{dw_x} = \frac{g_c}{\rho_s(1-\epsilon_x)} \frac{dp_s}{dx} = -\mu\alpha_x(q_x - e_x r_x) \tag{4}$$

When flow takes place through a fixed, compressible bed in which the solids are not moving, r_x is zero and q_x is constant (although the average liquid velocity q_x/ϵ_x may vary).

DIFFERENTIAL EQUATIONS FOR q_x AND r_x VARIATION

The basic differential equation relating the change in flow rate to the time rate of change of porosity has been presented in the form (*13*) of the equality of the first two terms in Equation (5).

$$\left(\frac{\partial q_x}{\partial x}\right)_\theta = \left(\frac{\partial \epsilon_x}{\partial \theta}\right)_x = -\left(\frac{\partial r_x}{\partial x}\right)_\theta \tag{5}$$

It will now be demonstrated that the last term in Equation (5) is valid. The total mass of dry solids per unit area between the septum ($x = 0$) and a distance x is given by

$$w_x = \rho_s \int_0^x (1-\epsilon_x)\, dx \tag{6}$$

As the average porosity in the distance zero to x decreases, solids must flow past the point x to replace the displaced liquid. A material balance over the cake from zero to x yields

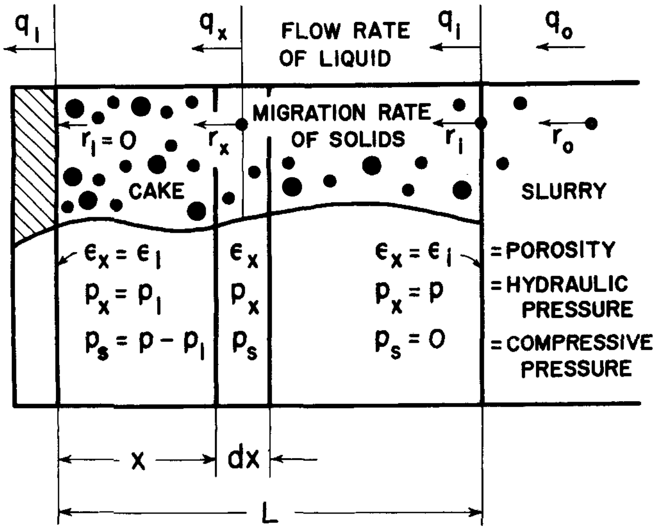

Fig. 1. Schematic diagram of cake.

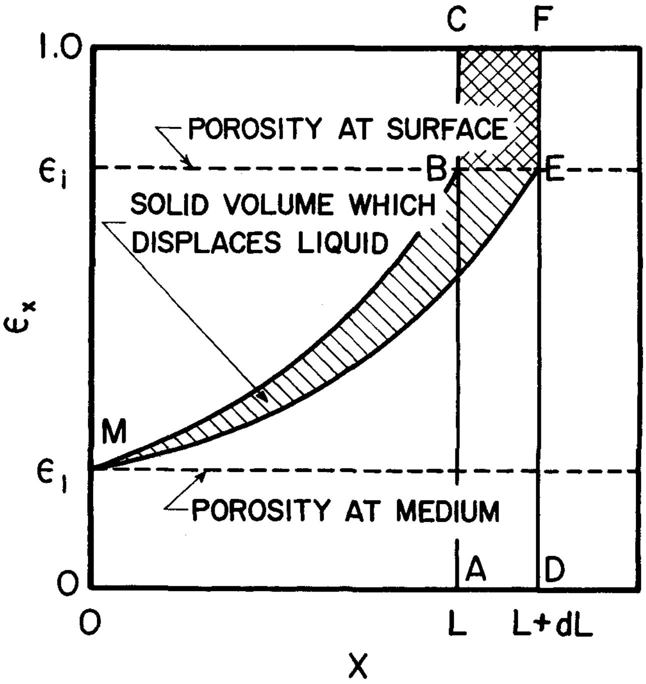

Fig. 2. Porosity vs. distance.

$$\rho_s r_x = \left(\frac{\partial w_x}{\partial \theta}\right)_x = -\rho_s \int_0^x \left(\frac{\partial \epsilon_x}{\partial \theta}\right)_x dx \tag{7}$$

provided there is no solids loss at the septum. The value of r_x gives the net flow of solids into the given volume. If there is loss of solid through the septum (cloudy filtrate), then Equation (7) would not be valid. Differentiating Equation (7) with respect to x gives

$$\left(\frac{\partial r_x}{\partial x}\right)_\theta = -\left(\frac{\partial \epsilon_x}{\partial \theta}\right)_x \tag{8}$$

thereby confirming Equation (5). Equation (5) leads to

$$dq_x + dr_x = 0 \tag{9}$$

which could be developed directly by considering the displacement of liquid by the moving solids. Integration of Equation (9) yields $q_x + r_x =$ constant; and evaluation of the constant at $x = 0$ where $r_x = 0$ and $q_x = q_1$ produces

$$q_x + r_x = q_1 \tag{10}$$

Equations (8) and (10) clearly indicate that variations in q_x are accompanied by corresponding variations in r_x. Tiller and Shirato (*14, 15*) utilized q_x variation in their works but failed to recognize that r_x variations were of equal importance. Their previous results will be modified.

Values of q_x and r_x at $x = L$

It is necessary to have formulae for the rate of flow q_i of the liquid and r_i of the solids at the interface of the cake and slurry. Material balances over a differential increase in thickness dL of the cake yield equations leading to values of q_i and r_i. Tiller and Cooper (*13*) derived an incorrect expression for q_i/q_1 which was later corrected (*15*).

For constant pressure filtration with negligible medium resistance in which the average porosity remains constant ($dm/d\theta = 0$) (*13, 15*), it has been shown that the flow rate q_1 at the medium is given by

$$q_1 = \frac{dv}{d\theta} = \frac{1-ms}{\rho s} \frac{dw}{d\theta} \tag{11}$$

At the cake surface where fresh solids are deposited, the apparent flow rate* q_o of liquid and r_o of solids approaching the cake surface may be presented as:

$$q_o = \frac{1-s}{\rho s}\frac{dw}{d\theta} \tag{12}$$

$$r_o = \frac{1}{\rho s}\frac{dw}{d\theta} \tag{13}$$

Since q_o and r_o refer to conditions approaching to the cake surface, the sum of q_o and r_o does not equal q_1 as indicated by Equation (10). If ϵ_i denotes the porosity in an infinitesimal surface layer of cake dL deposited in time $d\theta$, the solid volume which remains in the surface layer equals $(1 - \epsilon_i)dL$. Then one gets the apparent rate of flow of solids r_i at the cake surface as

$$r_i = r_o - (1-\epsilon_i)\frac{dL}{d\theta} \tag{14}$$

Since $w = \rho_s(1 - \epsilon_{avg.})L$ and $\epsilon_{avg.}$ is assumed constant, it is possible to eliminate L in Equation (14) and combine with Equation (13) to give

$$r_i = \frac{1}{\rho_s}\left[\frac{\epsilon_i - \epsilon_{avg.}}{1-\epsilon_{avg.}}\right]\frac{dw}{d\theta} \tag{15}$$

Dividing Equation (15) by (11) yields

$$\frac{r_i}{q_1} = \frac{\rho s(\epsilon_i - \epsilon_{avg.})}{\rho_s(1-\epsilon_{avg.})(1-ms)} = \frac{(\epsilon_i - \epsilon_{avg.})(m-1)}{\epsilon_{avg.}(1-ms)}s$$

$$= \frac{\rho s(\epsilon_i - \epsilon_{avg.})}{\rho_s(1-\epsilon_{avg.}) - s[\rho_s(1-\epsilon_{avg.}) + \rho\epsilon_{avg.}]} \tag{16}$$

If the cake is uniform $\epsilon_i = \epsilon_{avg.}$ or $s = 0$, then $r_i/q_1 = 0$. Previous derivations given in an erratum (*15*) to reference *13* report q_i/q_1 in the form

$$\frac{q_i}{q_1} = 1 - \frac{(\epsilon_i - \epsilon_{avg.})(m-1)}{\epsilon_{avg.}(1-ms)}s$$

$$= 1 - \frac{\rho s(\epsilon_i - \epsilon_{avg.})}{\rho_s(1-\epsilon_{avg.}) - s[\rho_s(1-\epsilon_{avg.}) + \rho\epsilon_{avg.}]} \tag{17}$$

Variations of r_x and q_x with distance x

Tiller and Shirato (*15*) showed that it was possible to assume $\epsilon_x = f(x/L)$ for constant pressure filtration with negligible medium resistance. Cake thickness L is a function of θ. Differentiating $\epsilon_x = f(x/L)$ with respect to time yields

$$\left(\frac{\partial\epsilon_x}{\partial\theta}\right)_x = \frac{d\epsilon_x}{d(x/L)}\frac{d(x/L)}{d\theta} = -\frac{x}{L^2}\frac{d\epsilon_x}{d(x/L)}\frac{dL}{d\theta} \tag{18}$$

Substituting Equation (18) into (7) and changing the limits from $(0, x)$ to (ϵ_1, ϵ_x) gives

$$r_x = \frac{dL}{d\theta}\int_{\epsilon_1}^{\epsilon_x}\left(\frac{x}{L}\right)d\epsilon_x \tag{19}$$

The value of r_i is obtained when $\epsilon_x = \epsilon_i$. Solving for r_x/r_i gives

$$\frac{r_x}{r_i} = \frac{\int_{\epsilon_1}^{\epsilon_x}(x/L)\,d\epsilon_x}{\int_{\epsilon_1}^{\epsilon_i}(x/L)\,d\epsilon_x} = \frac{(\epsilon_x - \epsilon_{avg.x})}{(\epsilon_i - \epsilon_{avg.})}\frac{x}{L} \tag{20}$$

where $\epsilon_{avg.x}$ is the average value of ϵ_x between zero and x. When $x/L = 1$, $\epsilon_x = \epsilon_i$ and $r_x/r_i = 1$; and when $x/L = 0$, $\epsilon_x = \epsilon_1$ and $r_x/r_i = 0$. Multiplying Equation (20) by (16) yields

$$\frac{r_x}{q_1} = \frac{(m-1)s}{\epsilon_{avg.}(1-ms)}(\epsilon_x - \epsilon_{avg.x})\frac{x}{L} \tag{21}$$

The apparent relative velocity of the liquid with respect to the solids divided by q_1 is given by

$$\frac{q_x - e_x r_x}{q_1} = \frac{q_x(1+e_x) - e_x q_1}{q_1} \tag{22}$$

At the medium where $r_x = 0$, the expression in Equation (22) is unity; and at the surface of the cake equals

$$\frac{q_i - e_i r_i}{q_1} = \frac{1-m_i s}{1-ms} \tag{23}$$

VARIATION OF p_x/p AND DEFINITION OF AVERAGE FILTRATION RESISTANCE

Integrating the basic differential equation [Equation (4)] through cake thickness x and the total thickness L and combining the results yields

$$\frac{p_x}{p} = \frac{\int_0^{x/L}\left[\left(\frac{q_x}{q_1}\right)(1+e_x) - e_x\right]\alpha_x(1-\epsilon_x)\,d(x/L)}{\int_0^{1}\left[\left(\frac{q_x}{q_1}\right)(1+e_x) - e_x\right]\alpha_x(1-\epsilon_x)\,d(x/L)} \tag{24}$$

A similar derivation in which r_x was neglected can be found in reference *15*. In deriving Equation (24) the filter medium resistance was neglected. The hydraulic pressure variation (p_x/p) *vs.* (x/L) can be obtained by utilizing Equation (24) in connection with compression-permeability experiments.

A new definition of average filtration resistance results when Equation (4) is utilized instead of Equation (1). The average filtration resistance α is generally given by

$$q_1 = \frac{dv}{d\theta} = \frac{g_c(p-p_1)}{\mu\alpha w} = \frac{g_c p}{\mu(\alpha w + R_m)} \tag{25}$$

where $g_c p_1 = \mu q_1 R_m$. Employing the first and last terms in Equation (4) and integrating across the entire cake yields

$$\int_0^w (q_x - e_x r_x)\,dw_x = \frac{g_c}{\mu}\int_{p_1}^{p}\frac{dp_x}{\alpha_x} = \frac{g_c}{\mu}\int_0^{p-p_1}\frac{dp_s}{\alpha_x} \tag{26}$$

The medium resistance is not neglected in Equation (26), and it is assumed that $dp_x = -dp_s$ and $p_x + p_s = p$. The first term in Equation (26) can be multiplied and divided by $q_1 w$ to give

$$\int_0^w (q_x - e_x r_x)\,dw_x = q_1 w\int_0^1\left(\frac{q_x}{q_1} - e_x\frac{r_x}{q_1}\right)d\left(\frac{w_x}{w}\right) \tag{27}$$

Solving for α in Equation (25) and substituting $q_1 w$ from Equation (27) leads to

$$\alpha = \frac{g_c(p-p_1)}{\mu q_1 w} = \int_0^1\left(\frac{q_x}{q_1} - e_x\frac{r_x}{q_1}\right)d\left(\frac{w_x}{w}\right)\left[\frac{p-p_1}{\int_0^{p-p_1}\frac{dp_s}{\alpha_x}}\right] \tag{28}$$

The value in brackets equals the conventional filtration resistance α_R as defined by Carman (*1*) and Ruth (*7*). Equation (28) may be written as

* It should be noted that both q_o and r_o are velocities relative to the cake surface and do not mean the velocities with reference to any fixed coordinate or to the medium.

$$\alpha = J_R \frac{p - p_1}{\int_o^{p-p_1} \frac{dp_s}{\alpha_x}} = J_R \alpha_R \qquad (29)$$

where J_R is the correction factor for α_R. For computational purposes J_R can be placed in the form

$$J_R = \int_o^1 \left[1 - \frac{(\epsilon_x - \epsilon_{avgx})(m-1)}{(1-\epsilon_x)\epsilon_{avg}(1-ms)} s\left(\frac{x}{L}\right)\right] d\left(\frac{w_x}{w}\right) \qquad (30)$$

As previously reported by Tiller and Shirato (*14, 15*), α_R is theoretically the average specific resistance of a compressible bed with the slurry concentration equal to zero. If s were actually zero, there would be no additional solids deposited. In practice, α_R is a good approximation of the average filtration resistance when the slurry is dilute. While too little information is available to generalize, it is probable that the effect of the solid movement can be neglected when the ratio of the fraction of solids in the slurry to the fraction in the surface layer of the cake is less than 0.5.

POROSITY AND FLOW VARIATIONS AS FUNCTIONS OF x/L

In Figure 3 calculations based on the equation of this paper illustrate the variations of (q_x/q_1), (r_x/q_1), and $(e_x r_x/q_1)$ with (w_x/w). Figure 3*a* corresponds to the case in which a slurry has a concentration less than its maximum possible value of $1/m_i$. The value m_i is the ratio of the mass of wet cake to the mass of dry cake in an infinitesimal surface layer. The reciprocal of m_i is simply the fraction of dry solids in the surface layer. When $s = 1/m_i$, the slurry reaches a solid state, and it is assumed that lower values of s will normally be encountered. Figure 3*b* corresponds to the case in which $s = 1/m_i$.

When $s = 1/m_i$, Equation (17) yields a value of $q_i/q_1 = \epsilon_i$, and $r_i/q_1 = 1 - \epsilon_i$. The relative velocity $(q_i - e_i r_i)$ at the cake surface is equal to zero. Therefore at the limiting condition for $s = 1/m_i$, the liquid and solid are moving with same velocities at the cake surface.

The difference between (q_x/q_1) and $(e_x r_x/q_1)$ represents the average relative velocity. The hatched area equals the value of J_R as defined by Equations (28), (29), and (30). The value of J_R is at its minimum in the case illustrated in Figure 3*b*, and J_R becomes unity when $s = 0$.

Calculations of rigorous internal flow variation based upon the equations in this paper require accurate data for filtration resistance as obtained from a compression-permeability cell in the form of α_x and ϵ_x as a function of the applied compressive pressure p_s. In the compression-permeability cell, there is an appreciable side friction between the wall and the cake. Most data found in the literature (*1* to *4*, *6* to *8*, *12*) have not taken wall friction into account and consequently do not yield strictly accurate values. The friction results in a portion of the applied load being absorbed in the wall with resulting lower average compressive pressure than the applied load at the top. Taylor (*11*) made a theoretical analysis of the load variation with depth on the basis of a simplified model. Calculations in the paper are based upon raw data obtained from compression-permeability cells. Further work needs to be done to relate wall friction to compression-permeability cell data.

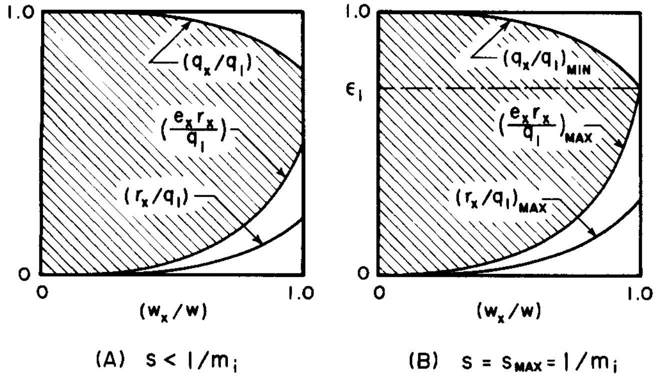

Fig. 3. q_x/q_1, r_x/q_1 and $e_x \cdot r_x/q_1$ vs. w_x/w.

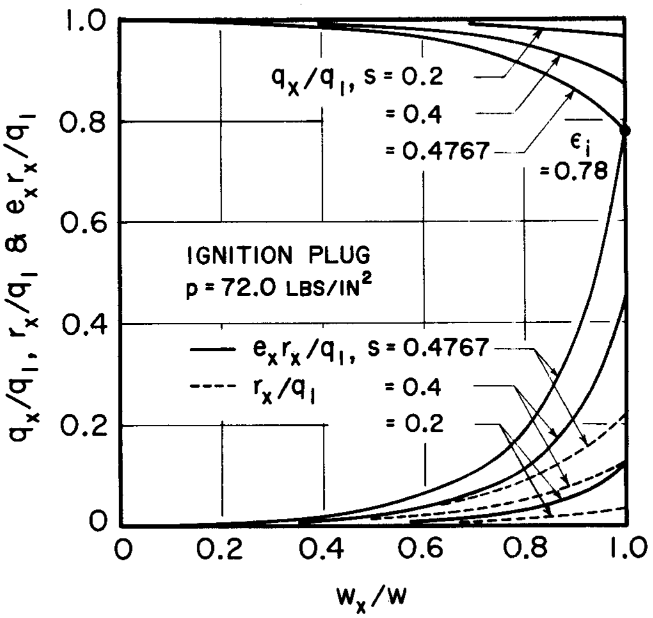

Fig. 4. q_x/q_1, r_x/q_1, and $e_x \cdot r_x/q_1$ vs. w_x/w (slurry for ignition plug).

Using raw porosity-pressure data obtained from a compression permeability cell and hydraulic pressure variation data (*6, 8, 15*) from actual cakes with tubes connected to air-sealed capillary manometers, values of (q_x/q_1), (r_x/q_1) and $(e_x r_x/q_1)$ were calculated for various slurry concentrations for ignition (spark) plug (alumina and clay) and cement material in Figures 4 and 5. The area between the q_x/q_1 and $e_x r_x/q_1$ curves gives the value of J_R.

It is apparent from Equation (30) that J_R depends upon filtration pressure and slurry concentration. While the pressure has relatively little effect, the value of J_R may change markedly for concentrated slurries of moderately compressible materials as illustrated in Figures 6 and 7. For ignition plug (alumina and clay) slurry filtered at 72.0 lbs/sq.in., the value of J_R is about 0.835 which is much smaller than the value reported by Tiller and Shirato (*14, 15*) on the basis of zero solid velocity.

CONCLUSION

It has been shown that the velocity of the solids may be

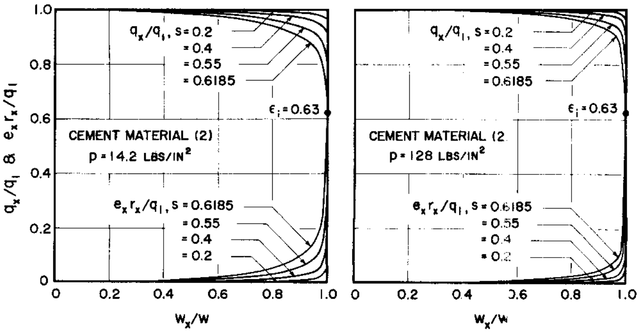

Fig. 5. q_x/q_1 and $e_x \cdot r_x/q_1$ vs. w_x/w [slurry for cement material (2)].

important in certain types of filter operations with concentrated slurries. A new definition of average filtration resistance should be more realistic than previous definitions which neglected movement of the solids.

ACKNOWLEDGMENT

The senior author is grateful to the National Science Foundation for support which contributed to this article. The authors acknowledge with gratitude the gifts of stainless steel pipes and plates by the Nippon Dyeing and Mfg. Company and of slurry materials by Kyoritsu Yogyo Genryo Company and Onoda Cement Company.

NOTATION

e_i = value of e_x in infinitesimal surface layer of cake, dimensionless
e_x = local value of voidage, $\epsilon_x/(1-\epsilon_x)$, dimensionless
g_c = conversion factor, poundal/pound force
J = correction factor previously defined by Tiller and Shirato (*15*), dimensionless
J_R = correction factor for the conventional Ruth's value α_R, dimensionless
L = total cake thickness, ft.
m = ratio of mass of wet to dry cake, dimensionless
m_i = value of m in infinitesimal surface layer of cake, dimensionless
p = applied filtration pressure, lb.-force/sq.ft.
p_s = cake compressive pressure at distance x from the medium, lb.-force/sq.ft.
p_x = hydraulic pressure at distance x from the medium, lb.-force/sq.ft.
p_1 = hydraulic pressure at the interface of medium and cake, lb.-force/sq.ft.
q_i = value of q_x in infinitesimal surface layer of cake, cu.ft./(sq.ft.)(sec.)
q_x = apparent flow rate of liquid at distance x from the medium, cu.ft./(sq.ft.)(sec.)
q_o = apparent rate of liquid flow approaching to cake surface, cu.ft./(sq.ft.)(sec.)
q_1 = value of q_x at the interface of medium and cake, cu.ft./(sq.ft.)(sec.)
r_i = value of r_x in infinitesimal surface layer of cake, cu.ft./(sq.ft.)(sec.)
r_x = apparent migration rate of solids at distance x from the medium, cu.ft./(sq.ft.)(sec.)
r_o = apparent rate of solid flow approaching to cake surface, cu.ft./(sq.ft.)(sec.)
r_1 = value of r_x at the interface of medium and cake, cu.ft./(sq.ft.)(sec.)

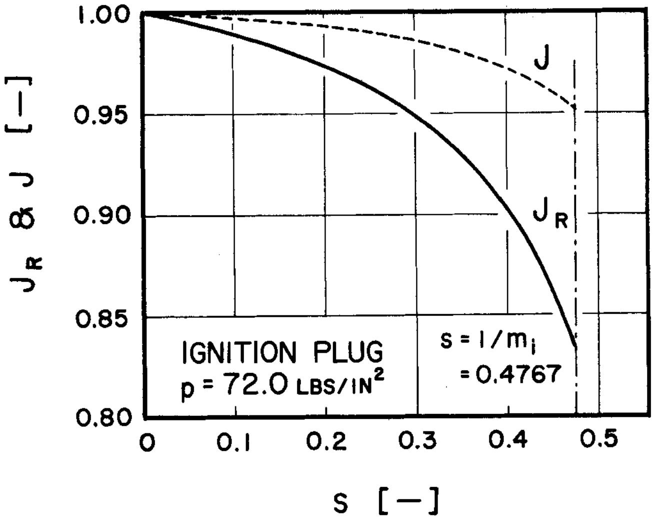

Fig. 6. Correction factors J_R and J vs. s (slurry for ignition plug).

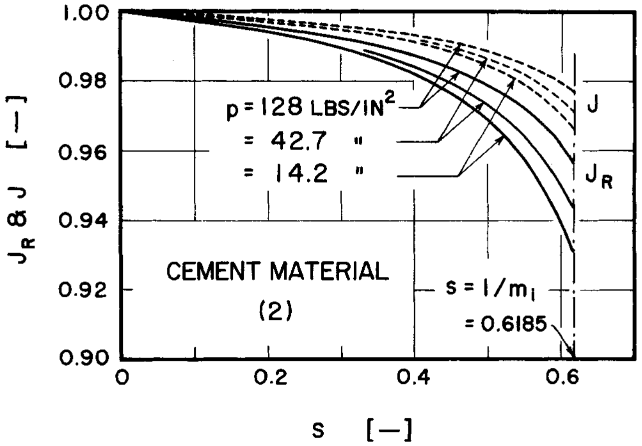

Fig. 7. J_R and J vs. s [slurry for cement material (2)].

s = mass fraction of solids in the slurry, dimensionless
u_x = true relative velocity of liquid to solids at distance x from the medium cu.ft./(sq.ft.)(sec.)
v = volume of filtrate per unit area, cu.ft./sq.ft.
w = mass of cake solids per unit area, lb.-mass/sq.ft.
w_x = mass of cake solids per unit area in distance x from the medium, lb.-mass/sq.ft.
x = distance from the medium, ft.

Greek Letters

α = average specific filtration resistance, ft./lb.-mass
α_R = average specific filtration resistance defined by Ruth, ft./lb.-mass
α_x = local value of specific filtration resistance at cake pressure p_s, ft./lb.-mass
ϵ_i = porosity in infinitesimal surface layer of cake, dimensionless
ϵ_x = local value of porosity at distance x from the medium, dimensionless
ϵ_{avg} = average porosity for entire cake, dimensionless
ϵ_{avgx} = average porosity for the portion of cake between medium and distance x, dimensionless
θ = time, sec.
μ = viscosity, lb.-mass/(ft.)(sec.)
ρ = density of liquid, lb.-mass/cu.ft.
ρ_s = true density of solids, lb.-mass/cu.ft.

LITERATURE CITED

1. Carman, P. C., *Trans. Inst. Chem. Engrs. (London)*, **16**, 168 (1938).
2. Grace, H. P., *Chem. Eng. Progr.*, **49**, 303, 367 (1953).
3. Hoffing, E. H., and F. J. Lockhart, *Ibid.*, **47**, 3 (1951).
4. Kottwitz, F. A., and D. R. Boylan, *AIChE J.*, **3**, 433 (1957).
5. Lu, W. M., MS thesis, Univer., Houston, Texas (1964).
6. Okamura, S., and M. Shirato, *Chem. Eng. (Japan)*, **19**, 104, 111 (1955), **20**, 98 (1956).
7. Ruth, B. F., *Ind. Eng. Chem.*, **38**, 564 (1946).
8. Shirato, M., and S. Okamura, *Chem. Eng. (Japan)*, **20**, 678 (1956), **23**, 11, 226 (1959).
9. ———, and M. Sambuichi, *Ibid.*, **27**, 470 (1963).
10. Shirato, M., M. Sambuichi, and H. Kato, Paper presented at the Second General Symposium, Soc. Chem. Engrs., Japan (1963).
11. Taylor, D. W., "Research on Consolidation of Clays," Mass. Inst. Tech., (Aug., 1942).
12. Tiller, F. M., *Chem. Eng. Progr.*, **49**, 467 (1953), **51**, 282 (1955).
13. Tiller, F. M., and H. R. Cooper, *AIChE J.*, **6**, 595 (1960).
14. Tiller, F. M., and M. Shirato, *Chem. Eng. (Japan)*, **26**, 925 (1962).
15. Tiller, F. M., and M. Shirato, *AIChE J.*, **10**, 61 (1964).

Manuscript received December 5, 1967; revision received March 15, 1968; paper accepted March 18, 1968.